

Probability and Counting Problems

by Andy Port for MAT-17C

- 1) How many 7 letter words can be spelled with the 7 letters abcdefg without repeating any letter?
- 2) How many 4 letter words can be spelled with the 7 letters abcdefg without repeating any letter?
- 3) How many 4 letter words can you spell with 2 A's and 2 B's?
- 4) When tossing 4 coins, what is the chance of getting exactly 2 heads?
- 5) When tossing 4 coins, what is the chance of getting at least 2 heads?
- 6) When rolling 2 dice, what is the chance of snake eyes (i.e. 2 ones)?
- 7) When rolling 2 dice, what is the chance of getting 1 one and 1 two?
- 8) When rolling 4 dice, what is the chance of getting 4 ones?
- 9) When rolling 4 dice, what is the chance of getting 2 ones and 2 threes?
- 10) An urn contains 6 blue balls and 4 green balls. Choosing 2 balls at random from the urn, what is the chance they are both red? Chance both blue? Chance different colors? What do these three probabilities add up to?
- 11) There are three boxes in front of you each containing four coins. One box contains all silver coins; one box contains all gold coins; one box contains half gold and half silver coins. You choose a box and take a coin from it. Given that this coin is gold, what is the probability of you reaching into the same box and pulling out another gold coin?
- 12) You are dealt 5 cards from a deck of 52 (13 cards of each suit). What is the chance you have a flush (i.e. all cards are of same suit)?
- 13) You are dealt 5 cards from a deck of 52 (4 cards of each value). What is the chance you have four of a kind (i.e. 4 cards of the same value)?
- 14) You are dealt 7 cards from a deck of 52 (4 cards of each value). The first two cards are an Ace and a King. What is the chance that you'll pick up a straight in the next five cards (i.e. receive a Queen, Jack and 10 each of which there are 4 in the deck)?

Answer Key

1) $7! = 5040$

2) $7!/3! = 840$

3) $\frac{4!}{2!2!} = 6$

4) $\frac{4!}{2!2!} \cdot \frac{1}{2^4} = 3/8$

5) $\frac{4!}{2!2!} \cdot \frac{1}{2^4} + \frac{4!}{1!3!} \cdot \frac{1}{2^4} + \frac{4!}{0!4!} \cdot \frac{1}{2^4} = 11/16$

6) $\frac{1}{6} \cdot \frac{1}{6} = 1/36$

7) $\frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = 1/18$

8) $\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = 1/1296$

9) $\frac{4!}{2!2!} \cdot \frac{1}{6^4} = 1/216$

10)

Both blue: $\frac{6}{10} \cdot \frac{5}{9} = \frac{1}{3}$

Both green: $\frac{4}{10} \cdot \frac{3}{9} = \frac{2}{15}$

Different colors: $\frac{6}{10} \cdot \frac{4}{9} + \frac{4}{10} \cdot \frac{6}{9} = \frac{8}{15}$

Sum: 1

11) $\frac{4}{6} \cdot \frac{4-1}{4-1} + \frac{2}{6} \cdot \frac{2-1}{4-1} = \frac{7}{9}$

12) 4 suits times $\binom{13}{5}$ different flushes per suit so... $\frac{4\binom{13}{5}}{\binom{52}{5}} = \frac{33}{16660} \approx 0.2\%$

13) 13 values (for our 4 of a kind) times $(52 - 4)$ choices for our fifth card so... $\frac{13 \cdot 48}{\binom{52}{5}} = \frac{1}{4165} \approx 0.02\%$

14) $4 \cdot 4 \cdot 4$ ways to pick our Q,J,10 times $\binom{52-5}{2}$ ways to pick the other two cards so...

$$\frac{4^3 \cdot \binom{47}{2}}{\binom{52}{5}} = \frac{4324}{162435} \approx 2.6\%$$