# 1 Summation Notation

When we wish to make a sum of many number, the following notation is used:

$$\sum_{i=1}^{n} f(i) := f(1) + f(2) + f(3) + \ldots + f(n-1) + f(n).$$

In summation notation, as this is called, the variable $i$ is an integer and the function f is evaluated at all integers between the lower and upper summation limits.

**Examples:**

1. $\sum_3^5 i^2 = 3^2 + 4^2 + 5^2 = 50$
2. $\sum_1^{10} i = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$
3. $\sum_1^3 1 + 1 + 1 = 3$
4. $\sum_0^2 sin(i\frac{\pi}{2}) = sin(0) + sin(\frac{\pi}{2}) + sin(\pi) = 1$

# 2 Summation Properties

| | |
|---|---|
| Constant | $\sum_{i=1}^{n} c = nc$ |
| Additivity | $\sum_{i=1}^{n} f(i) + g(i) = \sum_{i=1}^{n} f(i) + \sum_{i=1}^{n} g(i)$ |
| Linearity | $\sum_{i=1}^{n} af(i) + bg(i) = a\sum_{i=1}^{n} f(i) + b\sum_{i=1}^{n} g(i)$ |
| Constant Multiple | $\sum_{i=1}^{n} cf(i) = c\sum_{i=1} nf(i)$ |
| Summation Limits | $\sum_{i=a}^{b} f(i) + \sum_{i=b}^{c} f(i) = \sum_{i=a}^{c} f(i)$ |
| | $\sum_{i=a}^{b} c = (b-a+1)c$ |
| Monotonicity | If $f(i) \leq g(i)$ for each $i$ then $\sum_{i=a}^{b} f(i) \leq \sum_{i=a}^{b} g(i)$ |

**Examples:**

1. For $i \geq 3, i^2 \geq 9$, therefore$\sum_3^{10} 3 = 3(10-2) \leq \sum_3^{10} i^2$

# 3 Special Summations

| | |
|---|---|
| Constant | $\sum_{i=1}^{n} c = nc$ |
| | $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ |
| | $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ |
| | $\sum_{i=1}^{n} i^3 = (\frac{n(n+1)}{2})^2$ |

**Examples:**

1. $\sum_1^n 2i - 3i^2 = 2\sum_{i=1}^{n} i - 3\sum_{i=1}^{n} i^2 = 2\frac{n(n+1)}{2} - 3\frac{n(n+1)(2n+1)}{6}$

# 4 Area Computation by Regular Partitions

To find the area of the region bounded by the graph $y = f(x)$ (with $f(x) \geq 0$), the vertical lines $x = a$ and $x = b$ and the x-axis (that is, *the area under the curve* $y = f(x)$ *between* $a$ *and* $b$, proceed as follows:

1. Subdivide the interval $[a, b]$ into $n$ subintervals $[x_{i-1}, x_i]$, of equal width $\Delta x = \frac{b-a}{n}$. The endpoints $x_i = a + i\Delta x$.

2. In each interval, determine a point $x_i^*$ by a prescribed method. For example, for circumscribed rectangles choose $x_i^*$ equal to the point where the absolute maximum of $f$ occurs in the interval (assuming $f$ is continuous.)

3. Form the approximation to the area using the Riemann sum,

$$\sum_{i=1}^{i=n} f(x_i^*)\Delta x$$

and simplify using summation formulae.

4. Find the limit as $n$ "goes to infinity."

If $f$ is a continuous function, this limit exists and is called the *definite integral of* $f$ *from* $a$ *to* $b$ and denoted:

$$\int_a^b f$$

# 5 Exercise

Find the area under the curve $y = x^2 + x$ from $x = 1$ to $x = 2$ using the method of regular partitions and circumscribed rectangles.